

A BOHR'S SEMICLASSICAL MODEL OF THE BLACK HOLE THERMODYNAMICS

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Abstract

We propose a simple procedure for evaluating the main thermodynamical attributes of a Schwarzschild's black hole: Bekenstein-Hawking entropy, Hawking's temperature and Bekenstein's quantization of the surface area. We make use of the condition that the circumference of a great circle on the black hole horizon contains finite number of the corresponding reduced Compton's wavelength. It is essentially analogous to Bohr's quantization postulate in Bohr's atomic model interpreted by de Broglie's relation. We present black hole radiation in the form conceptually analogous to Bohr's postulate on the photon emission by discrete quantum jump of the electron within the Old quantum theory. It enables us, in accordance with Heisenberg's uncertainty relation and Bohr's correspondence principle, to make a rough estimate of the time interval for black hole evaporation, which turns out very close to

time interval predicted by the standard Hawking's theory. Our calculations confirm Bekenstein's semiclassical result for the energy quantization, in variance with Frasca's (2005) calculations. Finally we speculate about the possible source-energy distribution within the black hole horizon.

Key words: black hole entropy quantization - Hawking temperature

1 INTRODUCTION

Quantum theory, both the Old one and Quantum Mechanics, was designed to deal with microscopic phenomena, irrespective of the kind of the interaction involved. In practice, Bohr's, Heisenberg's and Schrödinger's theories deal almost exclusively with Coulombic interaction, as dominant at the atomic level. Strong and weak forces are restricted to nuclear and subnuclear levels and require a specific approach, outside the ordinary non relativistic quantum theory, partly because these interaction are difficult to describe by the potential functions. The fourth fundamental force, gravitations, has been left out, for a number of reasons. First, it appears so weak in comparison with the three ones mentioned, that at the microscopic level can be easily ignored. Second, it is the theory of gravitation, Newtonian, Einsteinian or else which is considered relevant to study gravitating bodies and celestial phenomena in general.

1.1 Newtonian and Coulombian systems

Though reigning at very different scales of the physical world Newtonian and Coulombian forces have a common formal structure which makes them attribute to the corresponding physical systems many common characteristic (see, e.g. Grujić 1993). These common features have been revealed in a particularly remarkable way when the Quantum Mechanics was formulated and a parallel with some General Relativity phenomena established.

The first modern hydrogen atom model was contrived by Thomson as the negatively charged electron immersed in a spherical positively charged fluid. This model was to be radically changed with the later Rutherford-Bohr model, but both had one remarkable feature in common: the path an electron traced while moving inside the fluid, or around the nucleus, was the same geometrical figure, an ellipse, despite the fact that electron experiences radically different forces. In the Thomson's model, the potential function is of the form

$$V(r) = kr^2 \quad (1)$$

that is as for the harmonic oscillator, whereas for the Coulombic interaction one has

$$V(r) = \beta/r \quad (2)$$

The difference was the positions of their foci. In the first case they were placed symmetrically with respect to the centre of the sphere, whereas in the motion around point-like nucleus the latter was positions at one of two foci. But the most remarkable similarity was revealed when comparing the semiclassical and quantum mechanical solutions of the corresponding energy spectra. It turns out that in both cases semiclassical and quantum mechanical results coincide, for all principal quantum numbers (so-called *correspondence identities*) (see, e.g. Norcliffe 1975). Hence, two most important interactions, harmonic oscillators and Coulombic ones allow for the semiclassical and quantum mechanical treatments indiscriminately. At the same time, it is for these interactions that any single-particle trajectory is closed, irrespective of the initial conditions. Not accidentally, these (textbook) interactions appear the only that allow for exact analytical solutions, both classical and quantum mechanical.

The energy spectra, evaluated in either of the theories, appear distinct, however. For the harmonic oscillator from Eq. (1) it reads

$$E_n = \hbar\omega(n + 3/2), n = 0, 1, 2, \dots \quad (3)$$

whereas the Coulombic case provides

$$E_n = \beta^2/2n^2, n = 1, 2, \dots \quad (4)$$

The first formula provides an equidistant distribution, whereas the Bohr's formula is the typical case for a series of discrete levels accumulating towards zero. It is this distinction which makes the investigation of the black hole spectrum of a particular interest, as we shall see below.

1.2 Black hole

It was only with the appearance of the concept of gravitational collapse and the model of black hole that the gravitational force becomes dominant and even exclusively present (see, e.g. Bekenstein 1994). In view of the formal similarity of the asymptotic behaviour of the Newtonian and Coulombic forces one may expect that the properties of atomic systems with charged constituents and black-hole like gravitational objects should share a number of common features. As noted by Bekenstein (1998) black hole is a hydrogen atom in the field of the strong gravity regime. In particular quantum effects may be present on the black hole surface and one may expect that some quantization rules are applicable.

One of the essential ingredients of the statistical mechanics has been the observation that the number of degrees of freedom of a quantum system should be proportional to the surface of the system, rather than to the volume. In fact, it was this assumption which led to the Bekenstein's linking of the black hole entropy and the area of its horizon.

Semiclassical quantization of black hole (BH) has been attempted by various authors. In a recent work Frasca (2005) calculated the semiclassical energy spectrum of the Schwarzschild black hole making use of the Hamilton-Jacobi formalism. For the stable circular orbits he derived the formula

$$E_n \approx M - \frac{2G^2 M^5}{n^2 \hbar^2} \quad (5)$$

which is, up to an additive constant M , Bohr's formula for the Coulombic interaction. In addition, the partition function turned out to coincide with that derived by the loop quantum gravity formalism (see, e.g. Nicolai *et al* 2005).

1.3 Black hole characteristics

Thermodynamical characteristics of a black hole is one of most important subject of the contemporary physics (see, e.g. very recent paper by Samuel and Chowdhury 2007). In a sense it plays the role of the black body studies around the turn of 19th century, linking the thermodynamics and statistics, more precisely the gravitation and information theories. First, Bekenstein (1973) suggested that a black hole contains the entropy S_{BH} proportional to the horizon surface area, A . For the Schwarzschild's black hole one has:

$$S_{BH} = \frac{k_B c^3}{4\hbar} A \quad (6)$$

where k_B is Boltzmann's constant, c speed of light and \hbar reduced Planck constant. Also, Bekenstein suggested that the horizon surface area is quantized, and can be changed only discretely

$$\Delta A = n 8 \frac{G\hbar}{c^3} \equiv n 8 L_P^2, n = 1, 2, . \quad (7)$$

where $L_P = (\frac{G\hbar}{c^3})^{\frac{1}{2}}$ is the Planck's length. Bekenstein's analysis is, on the one hand, based on the characteristics of corresponding, complex quantum measurement procedures, i.e. Heisenberg's uncertainty relations and Ehrenfest's adiabatic theorem. On the other hand it relies on general relativistic and quantum field theoretical requirement on the stability of the capture of a quantum system within black hole. According to this requirement, roughly speaking, Comptons wavelength of a given quantum system must be smaller than double Schwarzschild's radius. (Otherwise a quantum system can escape from black hole by means of quantum tunneling.)

After Bekenstein, Hawking (1975, 1976) showed that black hole can be considered as a black body which radiates at the temperature

$$T_H = \frac{\hbar c^3}{8\pi k_B G M} \quad (8)$$

where M is the black hole mass. This Hawking's temperature, according to usual rules of the thermodynamics, appears compatible to Bekenstein-Hawking entropy (6). Roughly speaking, Hawking's analysis is physically based on the non-invariance of the quantum field dynamics according to

general transformations of coordinates, which implies that a wave can be considered as a complex mixture of the plane waves (this mixture can be effectively treated as the spectrum of the black body). Simplified, black hole can gravitationally interact with fluctuated quantum vacuum near horizon. Then black hole can absorb one member of particle-antiparticle virtual pair, while other member of the pair can be effectively considered as the radiation. Mathematically, Hawking's analysis is based on a complex formalism of the quantum fields in the curved space (in a quasi-classical approximation). Later it has been proved, by Hawking (1979) and others, that Hawking's results can be reproduced even by more complex formalism, i.e. quantum field dynamics without quasi-classical approximations (see, for example, review articles (Wald 1997, 1999, Page 2004) and references therein).

Hawking, also, predicted time of the black hole evaporation. Namely, Stefan-Boltzmann law applied at the black hole surface area, according to Hawking's temperature (9) and relativistic equivalence relation $E = Mc^2$, has the form

$$-dE/dt = -c^2 dM/dt = \sigma_{SB} T_H^4 A = \frac{hc^6}{15360\pi G^2 M^2}, \quad (9)$$

where σ_{SB} is the Stefan-Boltzmann constant. It yields, after simple integration, the following expression for black hole evaporation

$$\tau_{ev} = 5120 \frac{\pi G^2}{\hbar c^4} M_0^3 \quad (10)$$

where M_0 denotes the black hole initial mass.

Further, detailed analysis of the quantum and thermodynamical characteristics of a black hole needs a very complex (in this moment incomplete) theoretical formalism including application of different string theories (Strominger and Vafa 1996, Proline 2006). Nevertheless, there are many attempts of the analysis of quantum and thermodynamical characteristics of a black hole by relatively simple (approximate) theoretical concepts. For example, in (Ram 2000, Ram *et al* 2005) it is shown that a black hole can be consistently considered as a Bose-Einstein condensate, while in (Nagatani 2007) a conceptual analogy between so-called minimum black hole and Bohrs model of the hydrogen atom is considered. Even in these cases mathematical formalism is based on different differ-

ential (e.g. Schrödinger's) equations solved within some (e.g. mean field) approximations.

2 THEORY

In this work we shall determine, in a simple way, three most important, thermodynamical characteristics of a Schwarzschild's black hole: Bekenstein-Hawking's entropy, Hawking's temperature and Bekenstein's quantization of the surface area. We shall use an original, simple and intuitively (quasi-classical) transparent condition. We demand that circumference of a great circle at black hole horizon contains integer (statistically averaged) number of corresponding reduced Comptons wavelength. It is essentially analogous to Bohrs quantization postulate in his Old quantum theory, interpreted by de Broglie's ontology, according to which circumference of an electron circular orbit comprises an integer number of corresponding de Broglie's wavelengths. Finally, we express the black hole radiation in the form conceptually analogous to Bohrs postulate on the photon emission by discrete quantum jump of the electron in his atomic model. It, in accordance with Heisenbergs energy-time uncertainty relation and a correspondence principle conceptually analogous to Bohr's one, admits a rough estimate of the time interval for black hole evaporation. This time interval is very close to the time interval of the black hole evaporation obtained via Hawking's radiation.

Thus, in this work we shall make a most simplified but non-trivial description of the quantum and thermodynamical characteristics of a Schwarzschild's black hole, which we simply call Bohr's black hole.

2.1 Bohr's black hole

Making use of de Broglie's relation

$$\lambda = \frac{h}{mv} \quad (11)$$

and Bohrs quantization postulate

$$mv_n r_n = n \frac{h}{2\pi}, n = 1, 2, \dots \quad (12)$$

it follows

$$2\pi r_n = n\lambda_n, n = 1, 2, \dots \quad (13)$$

where λ_n represents the n -th electron de Broglie's wavelength, m electron mass, v_n electron n -th speed, r_n - radius of the electron n -th circular orbit and h Planck constant. Expression (13) simply means that circumference of n -th electron circular orbit contains exactly n corresponding n -th de Broglie's wavelengths, for $n = 1, 2, \dots$

We shall now apply similar analysis of a Schwarzschild's black hole with mass M and Schwarzschild's radius

$$R_S = \frac{2GM}{c^2}. \quad (14)$$

We introduce the following expression analogous to (12)

$$m_n c R_S = n \frac{\hbar}{2\pi}, n = 1, 2, \dots \quad (15)$$

what implies

$$2\pi R_S = n \frac{\hbar}{m_n c}, n = 1, 2, \dots \quad (16)$$

analogous to (9). Here $2\pi R_S$ represents the circumference of the black hole while

$$\lambda_{rn} = \frac{\hbar}{m_n c}, n = 1, 2, \dots \quad (17)$$

is n -th reduced Compton's wavelength of a quantum system captured at the black hole horizon surface for expression (16) simply means that *circumference of the black hole horizon holds exactly n reduced Compton's wave lengths of a quantum system captured at the black hole horizon surface*. Obviously, it is essentially analogous to above mentioned Bohrs quantization postulate interpreted via de Broglie's relation. However, there is a principal difference with respect to Bohr's atomic model. Namely, in Bohr's atomic model different quantum numbers $n = 1, 2, \dots$, correspond to different circular orbits (with circumferences proportional to n^2). Here any quantum number $n = 1, 2, \dots$ corresponds to the same circular orbit (with circumference $2\pi R_S$).

According to (11) it follows

$$m_n = n \frac{\hbar}{2\pi c R_S} = n \frac{\hbar c}{4\pi G M} \equiv n m_1, n = 1, 2, \dots \quad (18)$$

where

$$m_1 = \frac{\hbar c}{4\pi GM} = \frac{M_P^2}{4\pi M} \quad (19)$$

and where $M_P = (\hbar c/G)^{1/2}$ is the Planck mass. Obviously, m_1 depends of M so that m_1 decreases when M increases and vice versa. For a macroscopic black hole, i.e. for $M \gg M_P$ it follows $m_1 \ll M_P$.

Suppose now that black hole mass equals

$$M = \sigma m_1 = \frac{\sigma \hbar}{4\pi c R_S} = \frac{\sigma \hbar c}{4\pi GM} \quad (20)$$

where σ denotes some integer (or approximately integer) number. According to (14,18) it follows

$$\sigma = \frac{M}{m_1} = \frac{4\pi GM^2}{\hbar c}. \quad (21)$$

It means that the number σ , for fixed black hole mass M , is finite.

After multiplication of (17) by Boltzmann constant k_B we have

$$k_B \sigma = \frac{4\pi k_B GM^2}{\hbar c} \quad (22)$$

Obviously, right-hand side of (18) represents Bekenstein-Hawking's entropy of the Schwarzschild's black hole (6). It is therefore reasonable to assume

$$S_{BH} = k_B \sigma = \frac{4\pi k_B GM^2}{\hbar c} \quad (23)$$

This assumption implies that σ must have a statistically appropriate form that will be considered later on.

Differentiation of (23) yields

$$dS_{BH} = k_B d\sigma = 8\pi k_B GM/(\hbar c^3) dE \quad (24)$$

where

$$E = Mc^2 \quad (25)$$

is the black hole energy. Expression (24), according to first thermodynamical law, implies that term

$$T = \hbar c^3 / 8\pi k_B GM = m_1 c^2 / (2k_B) \quad (26)$$

represents the black hole temperature. Evidently, this temperature is identical to Hawking's black hole temperature (8). According to (23), (24) it follows

$$dA = \frac{32\pi G^2}{c^3} M dM \quad (27)$$

or, in a corresponding finite difference form

$$\Delta A = \frac{32\pi G^2}{c^3} M \Delta M, \text{ for } \Delta M \ll M. \quad (28)$$

Further, we assume

$$\Delta M = m_n m_k = (n - k) m_1 = \frac{\hbar c}{4\pi G M}, n, k < n = 1, 2, \quad (29)$$

which, after substituting in (28), yields

$$\Delta A_{nk} = (n - k) 8 \frac{G \hbar}{c^3} = (n - k) 8 L_P^2, (n - k) = 1, 2, \dots \quad (30)$$

Obviously, expression (30) represents Bekenstein's quantization of the black hole horizon surface area (7).

In this way we have reproduced, i.e. determined in an independent way, three most important characteristics of Schwarzschild's black hole thermodynamics: Bekenstein-Hawking entropy, Hawking's temperature and Bekenstein's quantization of the surface area.

We now evaluate the necessary statistical form of σ . We suppose that black hole can be considered as a canonical statistical ensemble of Bose-Einstein quantum systems. Then the statistical sum, Z , according to (18), equals

$$Z = \sum_{n=0} \exp\left[-\frac{E_n}{k_B T_H}\right] \quad (31)$$

where

$$E_n = m_n c^2 = n \frac{\hbar c^3}{4\pi G M} = n \frac{M_P}{M} \frac{E_P}{4\pi} = n E_1, n = 0, 1, 2, \dots \quad (32)$$

and where $E_P = M_P c^2$ is Planck energy. (It is supposed, implicitly, that n can be zero. Or, precisely, it can be shown by a more detailed analysis, n can be changed by $(l(l+1))^{\frac{1}{2}}$ for $l = 0, 1, \dots$)

Hence, our calculations provide harmonic-oscillator-like spectrum, supporting Bekenstein's result, and in variance with Frasca's (2005) calculations. The difference between the latter two approaches concerns not only the mere spectrum of the black hole energy, but may shed the light onto the possible spatial energy-distribution within black hole. As described above, equidistant energy level distribution signals a uniform source-matter distribution, as the case of Thomson's atomic models shows. If the harmonic-oscillator-like spectrum proves correct, this would imply the uniformity of the gravitational field within the horizon.

According to (8), (33), it follows

$$\frac{E_1}{k_B T_H} = 2 \quad (33)$$

which introduced in (32) yields

$$Z = \sum_{n=0} \exp[-2n] = \exp[2]/(\exp[2] - 1) \quad (34)$$

Then

$$w_n = \exp[-\frac{E_n}{k_B T_H}]/Z = (\exp[2] - 1) \frac{\exp[-2n]}{\exp[2]} \quad (35)$$

represents probability of quantum (energy eigen) state n .

Further, it follows

$$M = -c^{-2} \frac{\partial(\ln[Z])}{\partial(1/(k_B T_H))} = \sum_{n=0} w_n m_n = \sum_{n=0} w_n n m_1 = m_1 \sum_{n=0} w_n n \quad (36)$$

which implies

$$\sigma = \sum_{n=0} w_n n = \langle N \rangle . \quad (37)$$

Obviously σ can be considered as the statistical average value $\langle N \rangle$ of the number of the quantum (energy eigen) states. On the other hand σ considered as statistically determined entropy (in k_B units) must have form

$$\sigma = - \sum_{n=0} w_n \ln[w_n] \quad (38)$$

Consistency of the analysis needs that (38) and (39) be equivalent which implies that condition

$$n = \ln[w_n], n = 0, 1, 2, \dots \quad (39)$$

must be satisfied. However, according to (35), it follows

$$\ln[w_n] = 2n \ln\left[\frac{\exp[2] - 1}{\exp[2]}\right] \approx 2n - 0.145, n = 0, 1, 2, \dots \quad (40)$$

what reveals that condition (39) is not satisfied. Nevertheless we note that left- and right-hand sides of (39) have the same order of magnitude, precisely that for large n right hand of (38) is twice greater than left-hand side of (41), what appears an interesting result.

We assume now that black hole represents a great statistical ensemble of Bose-Einstein systems with statistical sum

$$Z = \sum_{n=0} \exp\left[-\frac{E_n - \mu n}{k_B T_H}\right] \quad (41)$$

where μ represents the chemical potential while n in μn can be considered as statistical average value of Bose-Einstein systems in quantum (energy eigen) state n .

Suppose, further,

$$\mu = \frac{E_1}{2} \quad (42)$$

which, according to (32), implies

$$\mu n = \frac{E_n}{2}, n = 0, 1, 2, \dots \quad (43)$$

and, according to (32)

$$Z = \sum_{n=0} \exp\left[-\frac{E_n}{2k_B T_H}\right]. \quad (44)$$

In (44) Z can be considered as the statistical sum of a canonical ensemble with

$$w_n = \exp[-E_n/2k_B T_H](\exp[1] - 1) \exp[-n]/(Z \exp[1]), n = 0, 1, 2, \dots \quad (45)$$

It implies

$$\ln[w_n] = n \ln\left[\frac{\exp[1] - 1}{\exp[1]}\right] \approx n - 0.45, n = 0, 1, 2, \dots \quad (46)$$

and

$$\ln[w_n] \approx n, n \gg 1. \quad (47)$$

Relation (47) implies that condition (39) concerning the consistent statistical interpretation of σ is well satisfied for probabilities effectively defined by (46).

2.2 Energy spectrum and evaporation time

In Bohr's atomic model we have the postulate on the energy emission by discrete, spontaneous, quantum jump of the electron from a higher onto a lower circular orbit. This quantum jump represents an effective final result (or simplified description) of the electromagnetic self-interaction of the atom. Also, according to Bohr's correspondence principle, emission of the photon appears most probably by quantum jump of the electron from an initial, sufficiently high quantum state n onto the neighbouring final quantum state $(n - 1)$.

In conceptual analogy with Bohrs atomic model, suppose that black hole, considered as Bose-Einstein quantum system, in some initial quantum state n can spontaneously and discretely (by means of gravitational self-interaction) pass, i.e. jump, to some final, lower quantum state k , for $k < n = 1, 2, \dots$. Suppose, also, that by this quantum jump an effective final emission of a quantum of energy takes place which propagates far away from the black hole. Of course, black hole, according to its classical definition, captures any physical system near horizon by means of the gravitational interaction. Nevertheless, *according to principles of the quantum theory, (quantum mechanics and quantum field theory alike) gravitationally self-interacting black hole passes from an initially non-stable quantum state n in the final, stable quantum state $k < n$ by emitting one energy quantum outside horizon*. This is, of course, a simplified, phenomenological description of the black hole gravitational self-interaction.

Energy of given energy quantum, according to (32), equals

$$E_n - E_k = \hbar\omega_{nk}, k < n = 1, 2, \dots \quad (48)$$

where ω_{nk} is the circular frequency of given energy quantum. Then, according to (33), it follows

$$E_n - E_k = E_{n-k} = (n - k) \frac{c^3}{4\pi GM}, k < n = 1, 2, \dots \quad (49)$$

Here we assume that a correspondence principle, conceptually similar to Bohr's, holds. Precisely, suppose that for initial, large quantum state n , there is most probable quantum jump to the final state $k = n - 1$, with corresponding emission of the one energy quantum

$$E_n - E_{n-1} = E_1 = \frac{\hbar c^3}{4\pi GM}, n = 1, 2, \dots \quad (50)$$

Of course, given quantum jump can be considered definitive, i.e. irreversible, if and only if condition

$$\Delta E_n + \Delta E_{n-1} \ll E_n - E_{n-1} = E_1, n = 1, 2, \dots \quad (51)$$

is satisfied. Here ΔE_n and ΔE_{n-1} represent the energy natural widths of quantum states n and $n - 1$ and, for sufficiently large n we assume

$$\Delta E_n \approx \Delta E_{n-1} \quad (52)$$

For a more accurate form of (51) a more rigorous form of the quantum gravitation is necessary. Nevertheless, we shall simply suppose, according to (52),

$$2\Delta E_n \leq \frac{E_1}{100}, n \gg 1. \quad (53)$$

According to Heisenberg's energy-time uncertainty relation

$$\tau \Delta E_n \approx \frac{\hbar}{2}, n \gg 1, \quad (54)$$

where τ represents the time of the one energy quantum emission or life time of the Bose-Einstein system in the initial quantum state, it follows

$$\Delta E_n \approx \hbar/(2\tau), n \gg 1. \quad (55)$$

Then, according to (53), (55), it follows

$$t = 100\hbar/E_1 = 100 \cdot 4\pi GM/c^3, n \gg 1. \quad (56)$$

Suppose now that a black hole is initially in the (statistically averaged) quantum state $\frac{M}{m_1}$. Let the black hole, according to previous discussion, emit by quantum jump, energy quantum E_1 within time interval τ . It

implies that initial black hole with mass M will entirely evaporate by means of its gravitational self-interaction after a time interval τ_{ev} . Given time interval can be roughly estimated, according to (19), (32), by

$$\tau_{ev} \geq \frac{M}{m_1} 100 \frac{\hbar}{E_1} = 100(16\pi) \frac{\pi G^2 M^3}{c^4} \approx 5027 \frac{\pi G^2 M^3}{c^4}. \quad (57)$$

We see that the result is very close to Hawking's time for black hole evaporation (10).

3 CONCLUSION

We have carried out a simplified but non-trivial quasi-classical analysis of the quantum and thermodynamical characteristics of a Schwarzschild's black hole. Our analysis is conceptually analogous to formalism of Bohr's atomic model and for this reason black hole in our description can be simply called Bohr's black hole. We started by a condition, analogous to Bohr's quantization postulate, via de Broglie relation. This condition states that circumference of a great circle at black hole horizon contains an integer (statistically averaged) number of corresponding reduced Compton's wavelength. It implies simple determination of three most important thermodynamical characteristics of black hole: Bekenstein-Hawking entropy, Hawking's temperature and Bekenstein quantization of the surface area. Finally, we presented black hole radiation in the form conceptually analogous to Bohr's postulate on the photon emission by discrete quantum jump of the electron in Bohr's atomic model. It, in accordance with Heisenberg's energy-time uncertainty relation and a correspondence rule conceptually analogous to Bohr's correspondence principle, admits a rough estimate of the time interval for black hole evaporation. This time interval is very close to time interval of the black hole evaporation obtained via Hawking's radiation.

Finally, we have speculated about the relevance of the energy spectrum for the evidence of the source-field distribution within the horizon, which is, otherwise, unobservable quantity.

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BOROV SEMICLASICAN MODEL TERMODINAMIKE CRNE RUPE

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Predložen je jednostavan postupak za izračunavanje osnovnih termodinamičkih atributa Švarcšildove crne rupe: Bekenštajn-Hokingove entropije, Hokingove temperature i Bekenštajnovе kvantizacije površine horizonta. Korišćen je uslov da obim velikog kruga horizonta sadrži ceo broj redukovane Komptonove talasne dužine. Postupak je analogan Borovom postulat za kvantizaciju atoma vodonika preko de Brojjeve relacije. Postupak implicira uobičajeno značenje entropije crne rupe, u odnosu na površinu kvantne varijacije velikih krugova na horizontu. Zračenje crne rupe prezentirano je u obliku analognom Borovom konceptu emisije fotona putem diskrenih kvantnih skokova u okiru Stare Kvantne teorije. To omogućava, prema Hajzenbergovim relacijama neodređenosti i Borovom principu korespondencije, procenu vremenskog intervala za isparenje crne rupe, za koje je nadjeno da je veoma blisko intervalu prema standardnoj Hokingovoj formuli. Najzad, diskutovane su posledice izračunate energijske raspodele na procenu raspodele energije unutar crne rupe.